# Financial Intermediation and Regime Switching in Business Cycles

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*We study a one-sector growth model where capital investment is credit financed, and there is an adverse selection problem in credit markets. The presence of adverse selection creates an indeterminacy of equilibrium. Many equilibria display permanent fluctuations characterized by transitions between Walrasian regimes and regimes of credit rationing. Cyclical contractions involve declines in real interest rates, increases in credit rationing, and withdrawals of savings from banks. For some configurations of parameters all equilibria display cyclical fluctuations. We provide sufficient conditions for deterministic cycles consisting of m periods of expansion followed by n periods of contraction to exist.* (*JEL* E30, E32)

Monetary economists have frequently expressed the view that the financial system is an important source of—and propagation mechanism for—cyclical fluctuations. Indeed, John Maynard Keynes (1936), Henry Simons (1948), Milton Friedman (1960), and many others have argued that the free and unregulated operation of financial markets can lead to indeterminacy of equilibrium and "excessive economic fluctuations," even in the absence of shocks impinging on the rest of the economy. In modern terms, this argument claims that the financial system itself is a source of endogenously arising economic volatility.

This view has a strong empirical foundation. Most of the pre-World War II recessions were associated with substantial transfers of resources out of the banking system and into other assets. For instance, almost all of the pre-World War II recessions described by Friedman and Anna J. Schwartz (1963) were associated with increases in the currency-deposit ratio, and with an implied withdrawal of resources from the banking system. Particularly severe recessions were associated with particularly sharp increases in the currency-deposit ratio (that is, with bank panics). And even in the last three decades, several recessions have been accompanied by phenomena termed "disintermediation" or "credit crunches." In all of these episodes the volume of bank-extended credit declined, and "credit crunches" have often been associated with the increased incidence of nonprice rationing of credit.[1]

Why do we observe such sharp fluctuations in the volume of intermediated credit? Can these fluctuations be the "cause" of business cycles, as many (for instance, Friedman and Schwartz) have claimed, or are they merely a

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[1] See, for example, Stacey L. Schreft (1990) or Schreft and Raymond F. Owens (1995).

symptom of some more general phenomenon? If credit market activity does contribute to economic fluctuations, where does the impulse arise, and how is it transmitted to the real sector? And if the financial system is itself a source of endogenous shocks, are fluctuations in real economic activity merely a *possible* consequence or are they completely inevitable? For instance, can it be the case that under some circumstances the financial system *must* suffer from endogenous volatility which then spreads to the rest of the economy?

To answer these questions, this paper considers the relationship between credit and production in a simple model of dynamic general equilibrium, namely the nonmonetary overlapping generations economy with production introduced by Peter A. Diamond (1965).[2] We modify that economy in only three respects: we introduce some intragenerational heterogeneity, we force some kinds of capital investment to be credit financed, and we allow for the existence of an adverse selection problem in capital markets. Under complete public information about the characteristics of potential borrowers, we show that the first two of these modifications are purely cosmetic, and make no qualitative difference to the properties of competitive equilibria. Specifically, given any positive initial capital stock, the economy monotonically approaches a nontrivial steady-state equilibrium, which is unique under our assumptions. Thus equilibrium is determinate, and endogenous fluctuations cannot arise.

Heterogeneity, however, makes a lot of difference when there is private information about borrower characteristics (here *ex ante* information about loan repayment probabilities). In the presence of this adverse selection problem, lenders will seek to elicit from borrowers information regarding their loan repayment probabilities. Lenders will do so by structuring the loan contracts they offer—which specify both loan quantities and interest rates—to induce potential borrowers to self-select or, in effect, to reveal their type. Thus all loan contracts offered must be incentive compatible.

In our economy there is a range of values for the current capital stock (equivalently, for factor prices) which make the full-information allocation of credit incentive compatible. Here Walrasian allocations are competitive equilibria, and the equilibrium law of motion of the capital stock coincides with the law of motion that would prevail under perfect information. However, we also show that there is a range of current capital stocks for which Walrasian allocations cannot be incentive compatible; here incentive constraints must be binding and credit will be rationed. Credit rationing in our economy necessarily diminishes capital formation and reduces the level of real activity relative to what would be observed under public information. This raises two distinct possibilities. One is that the allocation of credit that would arise under public information is incentive compatible, or consistent with self-selection, under private information. In this case the full-information allocation can be duplicated by the appropriate choice of loan contracts, despite the presence of the informational asymmetry. As a result, a fully Walrasian allocation is feasible, and the adverse selection problem is innocuous. Alternatively, it may be impossible to induce self-selection without imposing quantity constraints on contracts. In this case the adverse selection problem "matters": lenders will use credit rationing as a means of separating borrowers with different *ex ante* repayment probabilities, and incentive constraints will be binding in equilibrium.

Finally, there is a nontrivial closed interval of current capital stocks having the following property: if the public-information allocation arises, it is incentive compatible and hence a true competitive equilibrium. At the same time, if credit rationing occurs, incentive constraints do bind on the choice of equilibrium loan contracts, so that a non-Walrasian equilibrium also exists in which the adverse selection problem does affect allocations. For this range of current capital stocks, then, the economy can be in either of two equilibrium regimes: a Walrasian regime in which competitive markets allocate credit in the standard way, or a regime of credit rationing. When

[2] Azariadis and Smith (1996a) study the role of fiat money or national debt in a neoclassical growth model with adverse selection.

credit is rationed, resources leave the banking system and the allocation of investment becomes less efficient in a manner we will make precise. Since each of these regimes is consistent with equilibrium, equilibrium is indeterminate: the economy can follow either the full-information or the private-information law of motion for the capital stock. Moreover, as we will demonstrate, there exist equilibria in which the economy can switch from one law of motion to the other in either a deterministic or a stochastic manner. These regime transitions will be associated with fluctuations in output and the capital stock which need not dampen over time. Thus both indeterminacy of equilibrium and "excessive" fluctuations can be observed when agents are privately informed about loan repayment probabilities.

Why does private information induce this regime switching, and the economic fluctuations that go with it? As we will show, Walrasian allocations are consistent with an incentive-compatible allocation of credit *only if the real rate of return on savings (deposits) is sufficiently high.* Then suppose the economy is currently in the Walrasian regime. If savers (depositors) anticipate a sufficiently low real return, they transfer some savings out of the banking system and into other, lower-yielding assets. This savings outflow forces banks to ration credit, firms to curtail capital investment, and real economic activity to decline. Moreover, the existence of credit rationing breaks the link between the marginal product of capital and the equilibrium rate of interest. This makes it possible for interest rates to fall in the way that savers anticipate. Thus depositors' expectations of declining interest rates become a self-fulfilling prophecy which forces a transition from the Walrasian regime into a regime of credit rationing. The consequence of tighter credit is an economic contraction in which both real interest rates and real activity decline.

Alternatively, suppose that the economy is currently in a regime of credit rationing. As we will demonstrate, *credit rationing can occur only if the real rate of return on savings (deposits) is sufficiently low.* If depositors then expect rising real interest rates, resources will flow back into the banking system. Credit can no longer be rationed, banks will finance more capital investment, and real activity will expand. Here changes in depositors' expectations act once more as a self-fulfilling prophecy of rising real interest rates, rising real activity, and a transition from a regime of credit rationing to one of Walrasian allocations.

As this discussion suggests, depositors' beliefs about financial regimes are at the heart of the multiplicity of equilibrium and of the regime transitions that can be observed here. This observation leads us to ask an additional question: are there any equilibria in which depositor beliefs never change? We describe some conditions under which the answer is yes. Under these conditions, there is necessarily a large set of equilibria. In two of these equilibria, depositor beliefs are invariant over time, and depositors always expect high (low) real interest rates. As a consequence, the economy is always in a Walrasian (credit-rationed) equilibrium. In addition, there is a large set of equilibria—which we characterize—where depositors' beliefs fluctuate in ways that induce regime transitions. Here it is possible to take the view that depositor beliefs fluctuate somewhat capriciously.

However, we also state conditions under which this turns out to be an overstatement, and there are *no* equilibria consistent with unchanging depositor expectations. When these conditions are satisfied, *all* equilibria display regime transitions and endogenously arising volatility.

Why are regime transitions a necessary feature of any equilibrium for some economies? As we have already argued, there is a lower bound on the rate of interest that is consistent with a Walrasian allocation. Moreover, in a Walrasian allocation, the real rate of interest and the marginal product of capital coincide. Thus, if the capital stock is too large (the marginal product of capital is too small) in a Walrasian allocation, this allocation cannot constitute an equilibrium for it is not consistent with an incentive-compatible allocation of credit. There is, then, an upper bound on the level of real activity that can be observed in the Walrasian regime.

Similarly, we show that there is an upper bound on the marginal product of capital (a lower bound on the level of real activity) that is consistent with the rationing of credit. If the capital stock is too low, banks will find bigger profits in lending indiscriminately to all borrowers at a common interest rate rather than separating them into "good" and "bad" risks. Therefore, rationing cannot occur if the capital stock is too low. This places a floor on the level of real economic activity.

Suppose that an economy is always in the Walrasian (credit-rationed) regime. Under our assumptions, the capital stock will monotonically approach its public-information (private-information) steady-state level. If this exceeds (is below) the upper (lower) bound on real activity that is consistent with a Walrasian (rationed) allocation of resources, then the implied time path of the economy is not an equilibrium outcome. Regime transitions must occur, and depositors must periodically revise their beliefs. Of course as this discussion suggests, if the full-information (private-information) steady-state capital stock is sufficiently low (high), there will be equilibrium paths where the same financial regime can prevail indefinitely. Here regime transitions may, but need not, occur.

For each of these cases, we examine the existence of deterministic perfect-foresight equilibria in which there are $m$ periods of expansion (in a Walrasian regime), followed by $n$ periods of contraction (in a regime of credit rationing). When it is possible to remain in each regime indefinitely, we show that such an equilibrium exists for *every* pair of integers $(m, n)$. Moreover, all such equilibria are asymptotically stable, so that the economy generates a high-dimensional indeterminacy of equilibrium. In addition, business cycles are potentially asymmetric—in the sense that expansions are longer (shorter) than contractions—if $m > (<) n$.

When it is *not* possible for either regime to prevail indefinitely, we are not guaranteed that cyclical equilibria exist for arbitrary $(m, n)$ pairs. In this case we describe conditions under which a deterministic equilibrium $(m, n)$ cycle does exist, for some pairs $(m, n)$. We also focus our attention on the following question: is there an equilibrium cycle along which depositor beliefs are revised as infrequently as possible? We call such equilibria *maximally persistent* $(m, n)$ cycles, and we state sufficient conditions for maximally persistent $(m, n)$ cycles to exist. We also show how to find the associated values of $m$ and $n$, and we indicate when expansions will be longer or shorter than contractions in a maximally persistent cycle.

The remainder of the paper proceeds as follows. Section I lays out the environment and the nature of trades, and describes the equilibrium conditions that obtain when credit is and is not rationed. Section II shows how the economy can transit between Walrasian regimes and regimes of credit rationing, while Sections III and IV examine the existence of cyclical equilibria which display undamped oscillations. Section V concludes.

## I. Capital Accumulation With Adverse Selection

### A. *Environment*

We consider a simple variant of Diamond's one-sector neoclassical growth model. In particular, at each date $t = 0, 1, \cdots$ a new set of two-period lived, overlapping generations is born. Each generation is identical in composition, and consists of a continuum of agents of measure one.

At each date there is a single consumption good, which is produced using a standard constant-returns-to-scale production function with capital and labor as inputs. A capital input of $K_t$ combined with a labor input of $L_t$ permits $F(K_t, L_t)$ units of this good to be produced at $t$. We let $k_t \equiv K_t/L_t$ denote the capital-labor ratio, and $f(k_t) \equiv F(k_t, 1)$ denote the intensive production function. We assume that $f(0) = 0$, that $f'(k) > 0 > f''(k)\ \forall k \geq 0$, and that $f$ satisfies the usual Inada conditions. We also assume that the consumption good can be stored: one unit of the good stored at $t$ returns $a$ units of the good at $t + 1$. Throughout we think of $a$ as being relatively small, so that storage is a relatively unproductive activity. Finally, the consumption good can be used to produce capital; one unit of consumption placed in a capital investment at $t$ returns one unit of capital at $t + 1$.

Within each generation, agents are divided into two types. A fraction $\gamma > 0.5$ of young agents are of type 1. These agents are endowed with one unit of labor when young, and are retired when old. In addition, they have access to the storage technology just described, but they have *no* access to the technology for converting current goods into future capital. A fraction $1 - \gamma < 0.5$ of each generation is of type 2. Type 2 agents cannot work when young, but are endowed with one unit of labor when old. In addition, we assume that type 2 agents have no access to the goods storage technology, but that they are endowed with the technology for converting current consumption into future capital. Thus, in many respects, type 1 and 2 agents are mirror images of one another. Finally, we assume that all agents care only about old-age consumption, and are risk neutral.[3] In particular, labor generates no disutility.

Notice that type 1 agents are natural lenders at any interest rate because they need to provide for old-age consumption. Type 2 agents are natural borrowers who need credit to finance capital investments. Between borrowers and lenders stand financial intermediaries, or banks, which accept deposits and extend loans. Their cost of doing so is zero.[4]

Finally, there is a set of initial old agents who are endowed with a per capita capital stock of $k_0 > 0$. We assume that capital depreciates completely in production.

## B. *Full Information*

In this subsection we describe a competitive equilibrium of this economy under the assumption of full information, and in particular that all allocations and the type of each agent are publicly observable. We also assume throughout, without loss of generality, that each type 2 agent (recall that these agents own capital) runs one firm and works for himself.

At date $t$ each young type 1 agent supplies one unit of labor inelastically, earning the real wage rate $w_t$. All of this income is saved, to be allocated between storage and bank deposits. We let $s_t$ be per capita storage at $t$, so $\gamma w_t - s_t$ is per capita deposits. Deposits earn the competitive gross return $R_{t+1}$ between $t$ and $t + 1$, which both banks and depositors treat as parametric.

Each young type 2 agent (firm) borrows $b_t$, and uses it to produce a per firm capital stock of $K_{t+1}$ at $t$. Loan market clearing, then, requires that

$$(1 - \gamma)b_t + s_t = \gamma w_t. \tag{1}$$

In addition, there will be a positive supply of deposits if and only if (iff) these weakly dominate storage in rate of return, so that

$$a \le R_t, \quad \forall t. \tag{2}$$

Old type 2 agents at $t$ have the inherited (per firm) capital stock $K_t$, which they combine with their own labor, plus $N_t$ units of young labor in order to produce output. In addition, these agents have an inherited interest obligation of $r_t b_{t-1} = r_t K_t$, where $r_t$ is the gross loan rate of interest, between $t - 1$ and $t$, charged by banks. Then old type 2 agents have an income (consumption) level of

$$F(K_t, 1 + N_t) - w_t N_t - r_t K_t,$$

which they maximize with respect to $N_t$. Hence if total labor input for the firm is $L_t = 1 + N_t$, we have

$$(3)\quad w_t = F_2(K_t, L_t)$$
$$\equiv f(k_t) - k_t f'(k_t) \equiv w(k_t); \quad t \ge 0.$$

Note that $w'(k) > 0$ holds for all $k$, and in addition we will assume that.

ASSUMPTION 1:

$$w''(k) < 0.$$

The above assumption holds if, for instance, $F$ is any constant-elasticity-of-substitution function with elasticity of substitution no less than one.

We assume that there is free entry into banking, so that

$$r_t = R_t \tag{4}$$

[3] The assumption that agents are risk neutral precludes lotteries from playing a useful role under private information. The assumption that agents care only about old-age consumption allows us to abstract from consumption/savings decisions. Both are inessential simplifications.

[4] It is not essential that credit extension be intermediated. However, the assumption that all credit is intermediated entails no loss of generality.

holds for all $t \geq 0$. In addition, competition among banks for borrowers implies that banks must offer type 2 agents at $t$ the loan quantity which maximizes their lifetime utility: $F(b_t, 1 + N_{t+1}) - w_{t+1}N_{t+1} - R_{t+1}b_t$. Hence

$$(5) \quad R_{t+1} = F_1(b_t, L_{t+1}) = F_1(K_{t+1}, L_{t+1}) = f'(k_{t+1}); \qquad t \geq 0.$$

Finally, since a fraction $1 - \gamma$ of the population is firms and $\gamma$ is workers at each date, clearly $N_t = \gamma/(1 - \gamma)$, and $L_t = 1 + N_t = 1/(1 - \gamma)$. Thus

$$(6) \quad k_t = K_t / L_t = (1 - \gamma)K_t = (1 - \gamma)b_{t-1}; \qquad t \geq 1.$$

It is now straightforward to describe the equilibrium law of motion for the capital stock. Equations (1), (3), and (6) imply that

$$(7) \quad k_{t+1} = \gamma w(k_t) - s_t.$$

Suppose that $f'(k_{t+1}) > a$. Then $s_t = 0$ holds, and (7) becomes

$$(7') \quad k_{t+1} = \gamma w(k_t).$$

Under our assumptions, (7′) describes an increasing, concave locus that passes through the origin, as depicted in Figure 1. If $\gamma w'(0) > 1$ holds, then there is a unique nontrivial steady-state equilibrium, denoted $k^0$ in Figure 1, Panel (a).[5] Given the initial per capita capital stock $k_0$, there is a unique sequence $\{k_t\}$ that monotonically converges to the steady state. Thus, under full information, this economy can display neither indeterminacy nor economic fluctuations.

### C. *Credit Rationing*

Next we introduce private information and examine what occurs when incentive constraints bind. Then, in Section II, we examine a full general equilibrium in which incentive constraints may or may not bind in an endogenous way.

We work with a simple structure of information; household type and storage activity are private information, while all market (both labor and credit market) transactions are publicly observed. These assumptions imply that young type 2 agents cannot credibly claim to be type 1 when young, since they are unable to supply labor. Type 1 agents, however, can borrow when young. If they do so, they must borrow the same amount ($b_t$) as type 2 agents, and they cannot work. In addition, type 1 agents are unable to produce capital, and hence cannot function as producers in old age. Thus any type 1 agent who borrows will be discovered as having misrepresented his type. In order to avoid punishment, a dissembling type 1 agent will simply store what he borrows and "go underground" or abscond with his loan. Thus a loan made to a type 1 agent will never be repaid.[6]

Given these circumstances, bank behavior must be modified in two respects. First, loan contracts must earn nonnegative profits. Let $\mu_t$ denote the fraction of type 1 agents who claim to be of type 2. Since such agents never repay their loans (and all borrowers borrow the same amount), banks earn nonnegative profits iff

$$(8) \quad r_{t+1} \geq R_{t+1}\{1 + \mu_t[\lambda/(1 - \lambda)]\}; \qquad t \geq 0.$$

Equation (8) requires that type 2 agents, who actually repay loans, pay enough to cover the

[5] This, of course, assumes that $f'(k^0) > a$.

[6] The hallmark of any model of credit rationing based on adverse selection or moral hazard [for example, Joseph Stiglitz and Andrew Weiss (1981) or Valerie R. Bencivenga and Smith (1993)] is that some borrowers have a higher probability of repayment than others, and hence care more about the interest rate dimension of the loan contract. Our specification is simply the most extreme (and simplest) version of this possibility: type 1 agents default on loans with probability one, and type 2 agents repay with the same probability. There is no conceptual difficulty associated with allowing type 1 and 2 agents to have loan repayment probabilities strictly between zero and one. However, this merely adds complexity without bringing any additional substantive issues into the analysis.

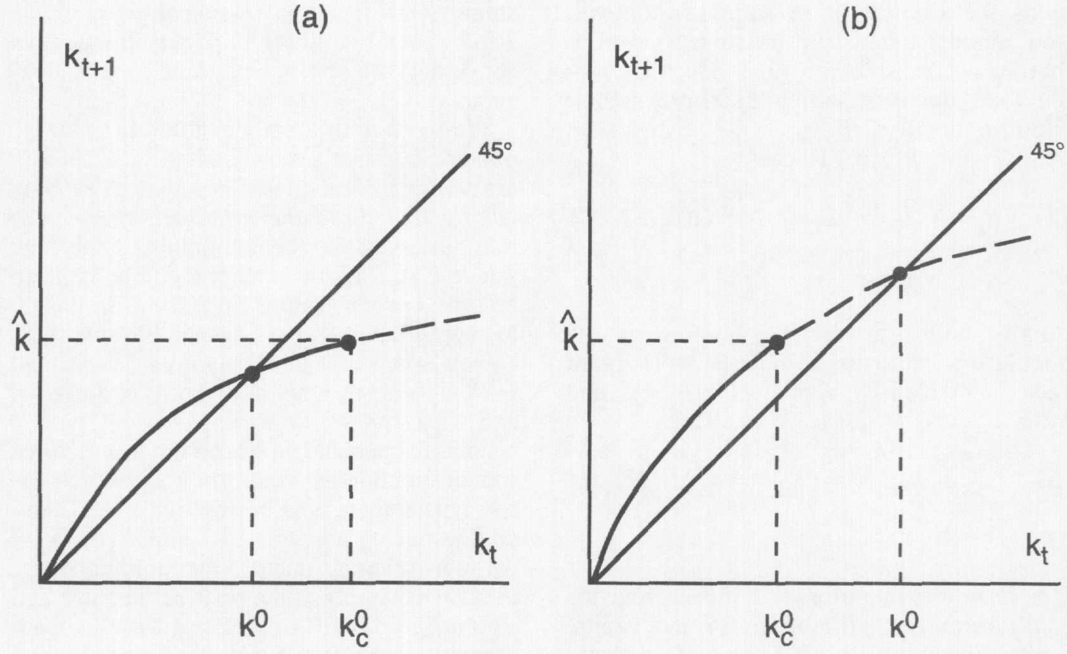


FIGURE 1. WALRASIAN EQUILIBRIUM LAW OF MOTION FOR $k_t$

defaults associated with loans to type 1 agents. Second, if $\mu_t < 1$ holds,[7] type 1 agents must do at least as well by working when young and saving as they would by claiming to be of type 2. Working when young and saving generates a lifetime utility level of $R_{t+1}w_t$ at $t$, while claiming to be of type 2 generates a lifetime utility level of $ab_t$. Hence loan contracts must satisfy the incentive constraint (or self-selection condition)[8]

$$(9) \qquad R_{t+1}w_t \geq ab_t; \qquad t \geq 0.$$

Since equation (9) is central to the analysis that follows, it is important to be clear about its content, and about the assumptions that support it. Equation (9) simply states the condition that is required in order to prevent type 1 agents from falsely misrepresenting their type, borrowing, and absconding with their loans. The essential assumptions underlying it are: (i) agents' types are private information; (ii) the amount borrowed is publicly observed, while (iii) the allocation of funds between investment and storage is not; and (iv) the returns on investment cannot be concealed, while the returns on storage can.

Assumption (i), of course, simply asserts that lenders confront an adverse selection problem, while assumption (ii) is quite standard in models of adverse selection. Assumption (iii) allows borrowers to divert funds from their "intended" uses; this is a common formulation in models of investment under

[7] If $\mu_t = 1$, no young agents work at $t$, no savings are supplied, and $k_{t+1} = 0$. The economy jumps to the autarkic steady state.

[8] Our assumption that labor-market activity is perfectly observable implies that dissembling type 1 agents can supply no labor when young. However, there is no conceptual difficulty with allowing dissembling type 1 agents to generate some labor income when young. In particular, suppose that a young type 1 agent who misrepresents his type can supply $\phi < 1$ units of labor, where $1 - \phi$ then represents the cost of working surreptitiously. Then equation (9) becomes $R_{t+1}w_t \geq a(b_t + \phi w_t)$, while equation (12) below must be replaced by $k_{t+1} = (1 - \gamma)(1 - \phi)w(k_t)$. Clearly this leads to no qualitative difference in results. We therefore pursue the simplest specification with $\phi = 0$.

moral hazard.[9] Assumption (iv) is based on the notion that some activities—like formal production utilizing a substantial capital input—are of a "public" nature and their outcome is difficult to conceal, while others are of a "private" nature which makes it easier to misappropriate their returns. Similar assumptions on return appropriability are common in models of moral hazard (Gertler and Hubbard, 1988; Gertler and Rogoff, 1990).

Following Michael Rothschild and Joseph Stiglitz (1976), we assume that intermediaries are Nash competitors in loan markets who take the deposit rate $R_t$ and the announced loan contracts of other banks as given. As in Rothschild and Stiglitz, it is easy to show that any Nash equilibrium contract earns zero profits, so that (8) holds with equality. In addition, it is possible to show that any nontrivial equilibrium (that is, any equilibrium with $k_t > 0\ \forall t$) has $\mu_t = 0\ \forall t$. In particular, contracts induce self-selection, and pooling is not a possibility.[10]

Thus, if an equilibrium with credit rationing exists, it continues to satisfy equations (3) and (6). Therefore (9) reduces to

$$(9') \qquad R_{t+1}w(k_t) \geq ak_{t+1}/(1-\gamma).$$

In addition, it is necessary that type 2 agents be willing to borrow; it is easy to show that they are if and only if (iff) the marginal product of capital exceeds the loan rate. In short, the inequality

$$(10) \qquad f'(k_{t+1}) \geq R_{t+1}$$

must hold. If (10) holds as a strict inequality, borrowers would like to borrow arbitrarily large amounts; in this case the incentive constraint (9) [or (9′)] holds as an equality and determines the equilibrium loan quantity. Thus, when credit rationing obtains, (9′) holds as an equality.

Finally, in order for announced loan contracts $(b_t, r_{t+1})$ to constitute a Nash equilibrium, it must be the case that no intermediary can offer an alternative contract $(\tilde{b}_t, \tilde{r}_{t+1})$ which is preferred by type 2 agents, and which satisfies (8) for some $\mu_t$. It is straightforward to show that, if there is such a contract, it must be a pooling contract ($\mu_t = 1$) with $\tilde{r}_{t+1} \geq R_{t+1}/(1-\gamma)$ in order to satisfy (8). Moreover, a pooling contract will attract type 2 agents iff $R_{t+1}/(1-\gamma) \geq f'(k_{t+1})$ holds.[11] Since it is impossible to observe a pooling contract in a nontrivial equilibrium, the existence of an equilibrium with credit rationing requires that there be no pooling contract which type 2 agents prefer to the contract $(b_t, R_{t+1}) = (R_{t+1}w_t/a, R_{t+1})$. This is so if

$$(11) \qquad f'(k_{t+1}) \leq R_{t+1}/(1-\gamma).$$

To summarize: in a regime of credit rationing, equations (3), (4), and (6) hold, as does (9′) at equality. Equations (10) and (11) must hold as well; these may (and typically will) be inequalities.

Prior to proceeding, it will be useful to have the following preliminary result. Its proof appears in Azariadis and Smith (1996b).

PROPOSITION 1: *Suppose that credit rationing occurs at t, and that $R_{t+1} > a$. Then*

$$k_{t+1} = \gamma w(k_t).$$

It is an immediate corollary of Proposition 1 that credit rationing has no bearing on the aggregate behavior of the economy unless $R_{t+1} = a$. We therefore focus our attention in the remainder of this section on equilibria in which there is credit rationing, and in which $R_{t+1} = a$ (real interest rates are low). The following proposition states our main result.

[9] See, for instance, Mark Gertler and R. Glenn Hubbard (1988), or Mark Gertler and Kenneth Rogoff (1990). Parenthetically, it is not essential to our results that type 1 agents are completely incapable of producing capital, or that type 2 agents are completely incapable of storing goods. All that is really required is that each type of agent is so inefficient in the appropriate activity that they are never tempted to undertake it.

[10] See Azariadis and Smith (1996a) for a proof in this particular context. Parenthetically, the argument they give depends on banks treating the deposit rate $R_t$ parametrically. See David Kreps (1990 Ch. 17) for the strategic foundations of this formulation.

[11] Again, see Azariadis and Smith (1996a) for a formal proof.

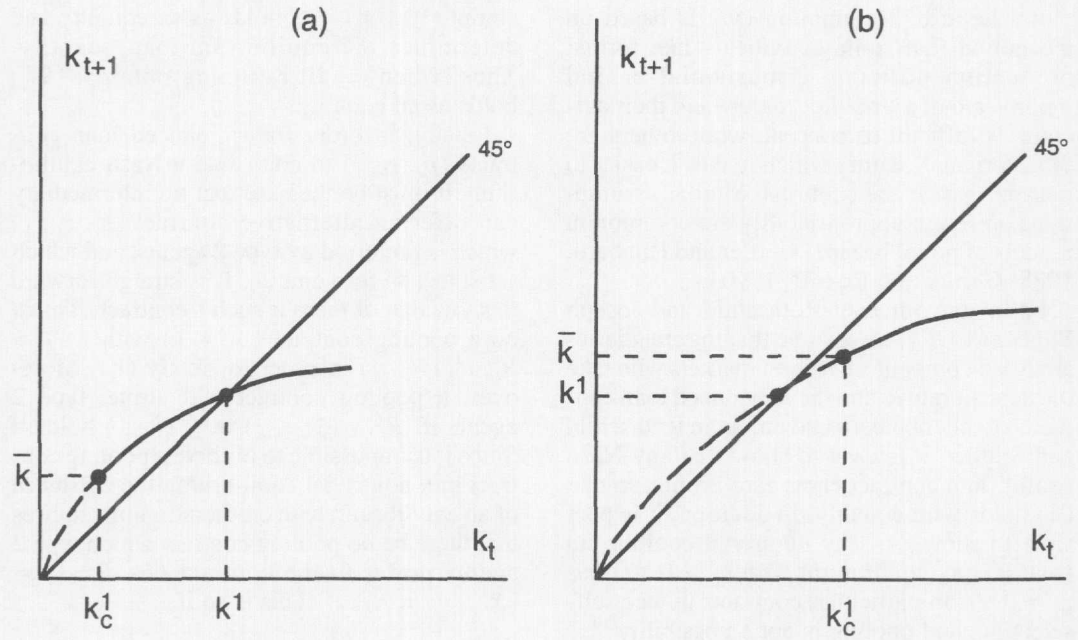


FIGURE 2. CREDIT RATIONING
LAW OF MOTION FOR $k_t$

PROPOSITION 2: *Suppose that there is credit rationing at t, and that $R_{t+1} = a$. Then the evolution of the capital stock is described by*

$$k_{t+1} = (1 - \gamma)w(k_t), \tag{12}$$

*while the aggregate (per capita) quantity of storage at t satisfies*

$$s_t = (2\gamma - 1)w(k_t) > 0. \tag{13}$$

Proposition 2 follows from substituting $R_{t+1} = a$ into (9′) to obtain (12), and from substituting (12) into (7), to obtain (13). The proposition asserts that, when credit rationing is accompanied by low real returns on savings, savings flow out of the banking system into the unintermediated activity we call "storage."

As noted in Proposition 2, when credit rationing does exist at date $t$, and when $R_{t+1} = a$ holds, equation (12) governs the evolution of the capital stock. Equation (12), of course, defines an increasing, concave function (as depicted in Figure 2) which has a unique nontrivial intersection with the 45-degree line iff $(1 - \gamma)w'(0) > 1$. We denote this intersection by $k^1$. Since $\gamma > 0.5$, the law of motion defined by (12) lies everywhere below the law of motion defined by (7)—so that credit rationing impedes capital formation—and $k^0 > k^1$ necessarily holds. Finally we note that, when credit is rationed, and when $R_{t+1} = a$, inequality (11) reduces to

$$f'(k_{t+1}) \le a/(1 - \gamma). \tag{14}$$

Thus (14) must hold at any $t$ for which there is credit rationing.

If credit is rationed at every date, then (12) gives the equilibrium sequence of capital stocks starting from $k_0$. Clearly this sequence will monotonically approach $k^1$, so that again equilibria are unique and display monotone dynamics. Multiple equilibria and endogenous fluctuations are associated with transitions between Walrasian regimes and regimes with credit rationing. Such transitions are the focus of the next section.

## II. Endogenous Regime-Switching Mechanisms

Suppose that, given the inherited capital-labor ratio $k_t$ at $t$, the value of $k_{t+1}$ yielded by (7′) satisfies $f'(k_{t+1}) > a$, and in addition that $(k_t, k_{t+1})$ satisfies (9′). Then the full-information allocation is incentive compatible, and constitutes an equilibrium at $t$. Alternatively, suppose that, given $k_t$, equation (9′) gives a value $k_{t+1}$ satisfying both $f'(k_{t+1}) \geq a$ and (14). Then credit rationing is consistent with an equilibrium outcome at $t$ in which $R_{t+1} = a$.

These two possibilities define two distinct regimes for each time period; in regime 0, the full-information allocation is incentive compatible and a Walrasian equilibrium obtains; in regime 1 the incentive constraint binds, $R_{t+1}$ equals $a$, and credit to borrowers is rationed. A brief description of each regime follows.

*Regime* 0—In order to describe when the economy can be in regime 0, it is useful to define two critical values for the capital stock. Let $\hat{k}$ satisfy

$$f'(\hat{k}) \equiv a\gamma/(1-\gamma), \tag{15a}$$

while $k_c^0$ is defined by

$$\hat{k} \equiv \gamma w(k_c^0). \tag{15b}$$

Then $\hat{k}$ is the largest "full-information capital stock" that can satisfy the incentive constraint, and $k_c^0$ is the capital stock that maps into $\hat{k}$ under (7′). These values are crucial ingredients in the analysis that follows; we now briefly elaborate on their significance.

Equations (7′) and (9′) indicate that the incentive constraint must be binding whenever the real rate of interest is too low. Thus large values of the capital stock are inconsistent with a Walrasian allocation because they imply excessively low interest rates. At these interest rates it is not possible to deter type 1 agents from misrepresenting their type without the existence of some credit rationing. Thus $k_{t+1}(k_t)$ cannot exceed $\hat{k}(k_c^0)$ without inducing some rationing of credit at time $t$.

Given the values of $(\hat{k}, k_c^0)$, it is easy to check that any sequence $(k_t, s_t, R_t)$ conforming to the initial condition $k_0$ is an equilibrium without credit rationing if, for each $t$,

$$k_{t+1} = \gamma w(k_t); \tag{16a}$$

$$s_t = 0; \tag{16b}$$

$$R_t = f'(k_t) > a; \tag{16c}$$

$$k_{t+1} < \hat{k} \text{ or } k_t < k_c^0. \tag{16d}$$

Equation (16) defines equilibrium sequences for which borrower characteristics are effectively public information. Solution sequences of (16a) converging to $k^0$ will be competitive equilibria for regime 0 if they are bounded above by the critical value $k_0^c$ defined in equations (15a) and (15b) and shown in Figure 1.

We conclude that it is feasible to be in a Walrasian regime if two conditions are met: (i) the level of economic activity is not too high (relative to $k_c^0$); and (ii) the investment projects available to low-quality borrowers (type 1 agents) are sufficiently less productive than those open to high-quality borrowers (type 2 agents).

*Regime* 1—Again, in order to describe when the economy can be in regime 1, it is necessary to define two more critical values of the capital stock. Let $\bar{k}$ and $k_c^1$ satisfy:

$$f'(\bar{k}) = a/(1-\gamma); \tag{17a}$$

$$\bar{k} = (1-\gamma)w(k_c^1), \tag{17b}$$

respectively. Then $\bar{k}$ is the smallest capital stock consistent with the existence of a separating Nash equilibrium under private information [see equation (14)], and $k_c^1$ is the capital stock that maps into $\bar{k}$ under (9′). From (15a) and (17a) we obtain

$$\bar{k} < \hat{k}. \tag{18}$$

As before, the values $\bar{k}$ and $k_c^1$ play an essential role in the subsequent analysis, so we again comment on their significance. Equation (14) indicates that, if the marginal product of capital is too high relative to the rate of interest (and $R_{t+1} = a$ holds in regime 1), it is not possible to deter lenders from pooling type 1 and type 2 agents, and lending to them indiscriminately. This outcome is inconsistent with the existence of a nontrivial equilibrium; hence the existence of an equilibrium within regime 1 requires that $k_{t+1} \geq \bar{k}$ ($k_t \geq k_c^1$) hold.

Given the values ($\bar{k}$, $k_c^1$), any sequence ($k_t$, $s_t$, $R_t$) that starts from a given initial condition $k_0$ is a competitive equilibrium with credit rationing if, for each $t$, it satisfies the following conditions:

$$k_{t+1} = (1 - \gamma)w(k_t); \tag{19a}$$

$$s_t = (2\gamma - 1)w(k_t); \tag{19b}$$

$$R_{t+1} = a < f'(k_{t+1}); \tag{19c}$$

$$k_{t+1} > \bar{k} \text{ or } k_t > k_c^1. \tag{19d}$$

As previously, the locus defined by (19a) in Figure 2 lies entirely below the locus defined by (16a), and its fixed point, $k^1$, lies below $k^0$.

Figure 2 also depicts the domain of definition, $[k_c^1, \infty)$, for equation (19a) and depicts two possible cases: the unique positive fixed point, $k^1$, of this equation is in that domain when Panel (a) obtains, but not when Panel (b) does. Given any initial value $k_0$ of the capital stock, separating competitive equilibria with credit rationing and $R_{t+1} = a$ are more likely to exist if the critical value $k_c^1$ is low relative to the fixed point $k^1$, i.e., if: (i) the level of economic activity is not too low (relative to $k_c^1$); and (ii) the investment projects available to low-quality borrowers are not much less productive than the ones open to high-quality borrowers.

### A. *Regime Transitions*

The previous section considered equilibria in which either regime 0 or regime 1 prevails permanently. However, it is also possible—or even necessary—that there exist equilibria in which the economy transits between these two regimes in either a deterministic, or a stochastic manner. This subsection discusses somewhat informally how and why an economy may shift from a Walrasian regime of slack incentive constraints to one of credit rationing and tight incentive constraints. Sections III and IV treat the same issues more formally and in greater depth. We start by amalgamating Figures 1 and 2 into the top two panels of Figure 3. In Figure 3(a) the fixed point of each regime is contained in its domain of definition; in other words, Figure 3(a) is drawn under the assumption that

$$k_c^1 < k^1 < k^0 < k_c^0. \tag{20a}$$

Figure 3(b), on the other hand, excludes the fixed point of each regime from its domain by assuming that

$$k^1 < k_c^1 < k_c^0 < k^0. \tag{20b}$$

We also draw in Panels (c) and (d) two other cases in which one of the fixed points is within the relevant domain of definition and the other is outside. Intuitively, inequality (20a) holds when the storage technology is much less productive than the neoclassical technology at $k^0$, while at $k^1$ banks are able to offer contracts separating high-quality borrowers from low-quality ones. Inequality (20b) on the other hand, means that the storage technology is only a little less productive than the neoclassical technology at $k^0$, and banks are completely unable to offer contracts permitting them to distinguish low- from high-quality borrowers at $k^1$.

Dynamic equilibria are solutions to the discontinuous, set-valued difference equation represented by the solid lines in each panel. To ensure that equilibria exist we require that the critical values ($k_c^0$, $k_c^1$) satisfy

$$k_c^1 < k_c^0. \tag{21}$$

Otherwise the solid graph in Panel (b) will contain a hole, and a deterministic $k_{t+1}$ will be undefined[12] if $k_t$ were to lie in the interval $[k_c^0, k_c^1]$. Azariadis and Smith (1996b) show that a necessary and sufficient condition for (21) is

$$\gamma\bar{k} < (1 - \gamma)\hat{k}, \tag{22}$$

where ($\hat{k}$, $\bar{k}$) are defined in equations (16a) and (18a).

[12] A stochastic equilibrium may exist for $k_{t+1}$ even if the deterministic map is undefined in some region. This solution may require mixed loan strategies from banks, e.g., stochastic credit rationing.

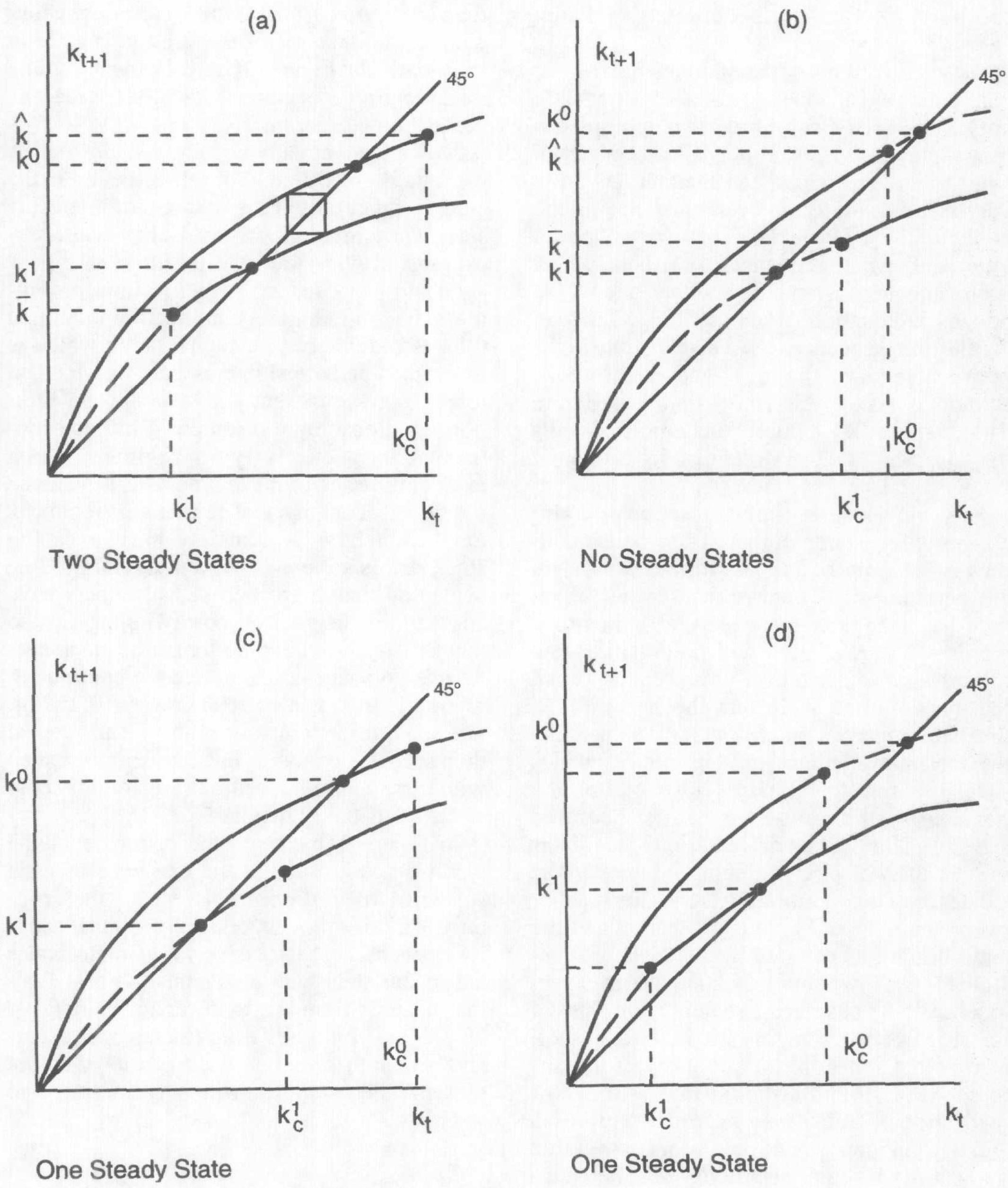


FIGURE 3. REGIME TRANSITIONS

Given the assumption that (22) holds, it is easy to see that each economy in Figure 3 has an *invariant set,* that is, a subset in its state space which traps all solution sequences that start in it. That set is the interval $[k^1, k^0]$ in Figure 3(a), and the interval $(\bar{k}, \hat{k})$ in Figure 3(b). Examples of sequences trapped within the former invariant set are regime 0 equilibria converging to $k^0$ from below, regime 1 equilibria converging to $k^1$ from above, as well as the periodic two-cycle depicted in Panel (a).

As we will show, in economies having the configuration of Panel (a), there is in fact a very large set of equilibria displaying deterministic cycles. Hence both indeterminacy of equilibrium and undamped fluctuations are a very real possibility. In economies having the configuration of Panel (b), there are no monotone equilibrium sequences $(k_t)$, because all such sequences eventually violate one of the bounds in equation (16d) or (19d). Thus all equilibrium sequences must display transitions between regimes 0 and 1, and therefore all economies having the configuration in Figure 3(b) *necessarily* exhibit fluctuations *despite* the absence of any variations in economic fundamentals.

How do these regime transitions occur? Suppose, for example, that at $t-1$ the economy is in regime 0. Suppose further that savers at $t$ pessimistically believe that the rate of return to savings will be low, or—in other words—that $R_{t+1} = a$ will hold. These low interest rate expectations imply that it is not feasible to have a Walrasian allocation at $t$, so that banks must ration credit. At the same time, the low rate of return on deposits persuades savers to transfer some savings out of the banking system, and into unintermediated storage. This ''disintermediation'' validates the necessity of credit rationing. Moreover, the existence of credit rationing breaks the link between the marginal product of capital and the equilibrium rate of interest: this makes it possible for the low returns that depositors expect to actually be observed in equilibrium. Thus a transition between regimes 0 and 1 is associated with a self-fulfilling prophecy of low interest rates, disintermediation, and credit restrictions. Transitions between regimes 1 and 0 occur similarly; if depositors expect that $R_{t+1} > a$ will hold, all savings will be channelled through the banking system, ruling out credit rationing. The equilibrium allocation must now be Walrasian. In short, then, switches between regimes 1 and 0 are accompanied by a self-fulfilling prophecy of rising interest rates and an availability of funds that allows the credit market to clear.

Figure 3(a) depicts a two-period cycle in which regime transitions occur despite the fact that economic fundamentals are consistent with a continuation of the present regime at each date. In Figure 3(b), however, regime transitions *must* occur periodically, for reasons that we now describe.

As we have already argued, a Walrasian allocation is consistent with self-selection in the credit market only if the real rate of interest is sufficiently high. (Excessively low real interest rates provide too little incentive to type 1 agents to work and save when young.) In Figure 3(b), an indefinite continuation in regime 0 leads ultimately to a capital stock which is too high—and a real interest rate which is too low—to be consistent with a Walrasian equilibrium allocation of resources. Thus an economy that is currently in regime 0 must eventually transit into regime 1. By the same token, an economy that remains in regime 1 long enough will ultimately have a capital stock that is so low—and a marginal product of capital that is so high—that lenders have an incentive to pool all borrowing agents, so that self-selection can no longer be sustained. At this point the economy cannot continue in regime 1, and must switch to regime 0. In Figure 3(b), these transitions must clearly repeat themselves. Economic fluctuations associated with these regime transitions are not only possible, but actually inevitable.[13]

In Figure 3(b), then, the operation of the credit market results in the existence of what we will term *reflective barriers* for the economy. An upper bound, or *ceiling*, on real activity exists because excessively low interest rates are inconsistent with a continuation of a Walrasian equilibrium. And a lower bound, or *floor*, on real activity arises from the fact that excessively high marginal products of capital do not present intermediaries with any incentive to

[13] A more realistic interpretation of this process, and one that would require a richer menu of borrowers than the one we are using, is to focus on the quality dispersion of loan applicants through the business cycle. In a business expansion, poorly managed firms expand along with better firms, and poor investment projects become marginally profitable. The resulting high dispersion in the quality of loan applications raises the riskiness of intermediary loan portfolios. Banks eventually respond by rationing credit because they lack accurate information about the risk characteristics of individual projects.

ration credit. When these reflective barriers satisfy (20b), real activity will necessarily fluctuate even if economic fundamentals do not vary over time. In these fluctuations, downturns (upturns) are associated with falling (rising) real interest rates, with increased (reduced) rationing of credit, and with resources leaving (reentering) the banking system.

As this discussion suggests, it is important whether or not the positive steady states $(k^1, k^0)$ of the two regimes fall outside the interval $(k_c^1, k_c^0)$ defined by the reflective barriers. Accordingly, we arrange our investigation around reflective barriers: we study economies with no binding barriers in Section III, and economies with two binding barriers in Section IV. These discussions will also illustrate what would happen in economies with the configurations depicted in Figures 3(c) and 3(d).

### III. Economies with Steady-State Equilibria

When equation (20a) is satisfied, equilibrium sequences $(k_t)$ evolve according to the set-valued difference equation

$$(23)\quad k_{t+1} = \gamma w(k_t); \qquad k_t < k_c^1$$

$$k_{t+1} \in \{\gamma w(k_t), (1-\gamma)w(k_t)\};$$
$$k_c^1 \le k_t \le k_c^0;$$

$$(24)\quad k_{t+1} = (1-\gamma)w(k_t); \qquad k_t > k_c^0.$$

We are particularly interested in periodic solutions, that is, in fixed points of iterated maps derived by repeated application of (23) and (24) to describe how today's state variable is related to its value $n = 1, 2, 3, \ldots$, periods hence. Because the map is set valued, iterates of a given order $n$ depend very much on which branch is chosen at each iteration. For instance, always choosing the lower branch $(1-\gamma)w$ of the map will produce a smaller iterate than if we always choose the upper branch $\gamma w$.

To simplify the mathematical structure, we endow the economy with a logarithmic production function which reduces the maps in (23) and (24) to a piecewise linear difference equation in the logarithm of the capital stock. We assume, in particular, that

$$(25)\quad f(k) = (A/\theta)k^\theta;$$
$$A > 0, \qquad \theta \in (0,1).$$

Defining $x = \log k$, we may rewrite equations (23) and (24) in their linear form:

$$(26)\quad x_{t+1} = f_0(x_t) \equiv \theta(x_t - x^0) + x^0;$$
$$x_t < x_c^1$$

$$x_{t+1} \in \{f_0(x_t), f_1(x_t)\}; \qquad x_c^1 \le x_t \le x_c^0$$

$$x_{t+1} = f_1(x_t) \equiv \theta(x_t - x^1) + x^1; \qquad x_t > x_c^0,$$

where

$$(27)\quad x^0 = [1/(1-\theta)]\log[\gamma(1-\theta)A/\theta]$$
$$\equiv \log k^0;$$

$$(28)\quad x^1 = [1/(1-\theta)]$$
$$\times \log[(1-\gamma)(1-\theta)A/\theta]$$
$$\equiv \log k^1$$

are the fixed points of regimes 0 and 1, respectively. It is also useful to compute the critical points for the two regimes. Under the technological assumption in equation (25), these are given by:

$$(29\text{a})\quad x_c^0 = (1/\theta)\log\left\{\left[\frac{\theta}{\gamma(1-\theta)A}\right]\times\left[\frac{A(1-\gamma)}{a\gamma}\right]^{1/(1-\theta)}\right\}$$
$$\equiv \log k_c^0;$$

$$(29\text{b})\quad x_c^1 = (1/\theta)\log\left\{\left[\frac{\theta}{(1-\gamma)(1-\theta)A}\right]\times\left[\frac{A(1-\gamma)}{a}\right]^{1/(1-\theta)}\right\}$$
$$\equiv \log k_c^1.$$

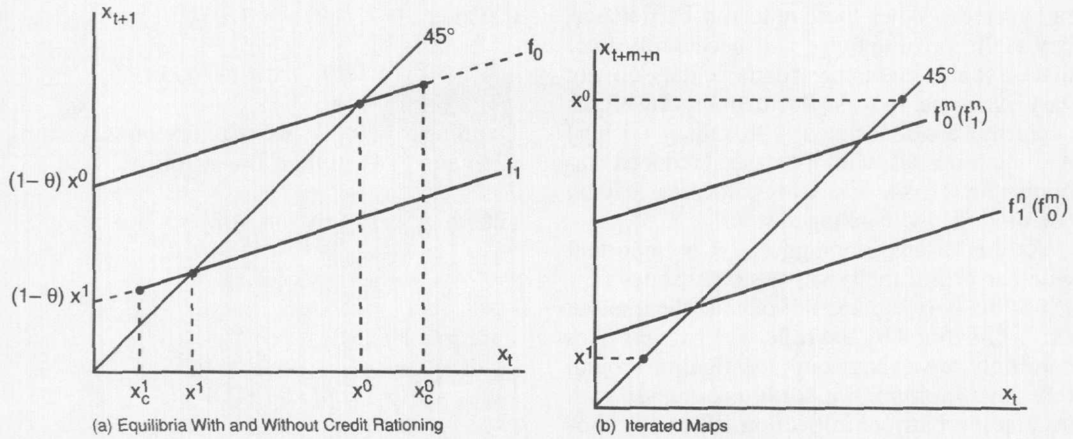


FIGURE 4. EQUILIBRIUM LAW OF MOTION (COBB-DOUGLAS PRODUCTION)

The map defined by (26) is depicted in Figure 4(a). In order to ensure that this map contains no holes and that deterministic equilibria exist, we assume $x_c^1 < x_c^0$ or, equivalently,

$$(30)\qquad 1 > \gamma\left(\frac{\gamma}{1-\gamma}\right)^{1-\theta}.$$

In addition, capital dominates storage in rate of return for all $k \leq k^0$ iff $f'(k^0) > a$ holds. This condition proves to be equivalent to the restriction

$$(31)\qquad \gamma < \frac{\theta/(1-\theta)}{\max\{0, a\}}.$$

Finally, an economy has no effective reflective barriers if the map (26) contains in its domain of definition the fixed points $(x^0, x^1)$ of each regime. This inclusion means that

$$(32)\qquad x_c^1 < x^1 < x^0 < x_c^0$$

or, equivalently, that

$$(33)\qquad a\gamma^2/(1-\gamma) < \theta/(1-\theta) < a.$$

In the remainder of this section we study solutions to equation (26), maintaining inequalities (30)–(33) as restrictions on the economy's parameter space $(a, \gamma, \theta, A)$. Inequality (32), in particular, implies that the steady state of each regime is an equilibrium and, hence, regime transitions *need not* occur in equilibrium. If there are switches in regimes, this will be due to self-confirming expectations by depositors about the behavior of intermediaries and the yields on bank liabilities.

We are now prepared to establish the main result of this section, which indicates that this economy can display very strong indeterminacies and excessive fluctuations in perfect-foresight equilibria. It is an almost immediate implication that there is also a wide variety of equilibria where the economy experiences stochastic shifts between regimes 0 and 1 in a Markovian manner, with the probability of regime transitions depending potentially on time, history, or the state of the system.

To begin, it will be useful to orient our formal discussion around the notion of "an $(m, n)$ cycle." Loosely speaking, an $(m, n)$ cycle is defined to be a deterministic cycle of $m$ periods spent in regime 0, followed by $n$ periods spent in regime 1 (and so on). In order to define an $(m, n)$ cycle more formally, we begin by considering compound iterates of the maps $f_0$ and $f_1$ introduced in equation (26). In particular, let $f_0^m(f_1^n)$ denote the map $f_0(f_1)$ iterated with itself $m(n)$ times. Then:

$$(34a)\qquad f_0^m(x) = \theta^m x + (1-\theta^m)x^0;$$

$$(34b)\qquad f_1^n(x) = \theta^n x + (1-\theta^n)x^1.$$

Compound iterates of the form $f_0^m[f_1^n(x)]$ and $(f_1^n[f_0^m(x)])$ describe orbits that spend $n$ periods in regime 1 ($m$ periods in regime 0) followed by $m$ periods in regime 0 ($n$ periods in regime 1). For given $(m, n)$, these iterated maps satisfy

$$(34c)\quad f_0^m[f_1^n(x)] = \theta^{m+n}x + (1-\theta^m)x^0 + \theta^m(1-\theta^n)x^1;$$

$$(34d)\quad f_1^n[f_0^m(x)] = \theta^{m+n}x + \theta^n(1-\theta^m)x^0 + (1-\theta^n)x^1.$$

Figure 4(b) graphs these maps. Since $x^0 > x^1$, we have

$$(35)\quad f_0^m(f_1^n) > f_1^n(f_0^m) \qquad \forall x.$$

Finally, we may compute two types of double compound iterates that change regime twice. One starts with $m-q$ periods in regime 0, continues with $n$ periods in regime 1, and follows with $q$ periods in regime 0 (and so on). The second starts with $n-p$ periods in regime 1, followed by $m$ periods in regime 0, and ending with $p$ periods in regime 1 again (and so on). Maps that describe such orbits are given by

$$(36a)\quad f_0^q[f_1^n(f_0^{m-q}(x))] = \theta^{m+n}x + [\theta^{n+q}(1-\theta^{m-q}) + (1-\theta^q)]x^0 + \theta^q(1-\theta^n)x^1$$

and

$$(36b)\quad f_1^p[f_0^n(f_1^{n-p}(x))] = \theta^{m+n}x + [\theta^{m+p}(1-\theta^{n-p}) + (1-\theta^p)]x^1 + \theta^p(1-\theta^m)x^0,$$

respectively. Then we have the following definition.

*Definition:* For given $m \geq 1$ and $n \geq 1$, the sequence $(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_m, \tilde{x}_{m+1}, \ldots, \tilde{x}_{m+n})$ constitutes an $(m, n)$ cycle if

(i) $\tilde{x}_i \in [x^1, x^0]$, $\forall i = 1, \ldots, m+n$, and either

(ii) $\tilde{x}_i = f_0^{i-1} \circ f_1^n \circ f_0^{m+1-i}(\tilde{x}_i)$, $i = 1, \ldots, m$,[14] or

(iii) $\tilde{x}_i = f_1^{i-m-1} \circ f_0^m \circ f_1^{n+m+1-i}(\tilde{x}_i)$ for $i = m+1, \ldots, m+n$.

Thus the values $\tilde{x}_i$ are each fixed points of the appropriate compound maps.

We are now prepared to state the following proposition. Its proof appears in Azariadis and Smith (1996b).

PROPOSITION 3: *(a) For all positive integers m and n, there exists an asymptotically stable (m, n) cycle; (b) Any (m, n) cycle has the property that* $\tilde{x}_1 \leq \tilde{x}_i \leq \tilde{x}_{m+1}$, $\forall i = 1, \ldots, m+1$. *Moreover,*

$$(37)\quad \tilde{x}_1 = [1-\Gamma(m,n)]x^0 + \Gamma(m,n)x^1;$$

$$(38)\quad \tilde{x}_{m+1} = [1-\theta^m\Gamma(m,n)]x^0 + \theta^m\Gamma(m,n)x^1,$$

*where*

$$(39)\quad \Gamma(m,n) \equiv (1-\theta^n)/(1-\theta^{m+n}).$$

These equations imply that, for any $(m, n)$, both of the extreme periodic points $\tilde{x}_1$ and $\tilde{x}_{m+1}$ lie within the invariant interval $[x^0, x^1]$; hence the remaining fixed points $(\tilde{x}_2, \ldots, \tilde{x}_{m-1}, \tilde{x}_m, \ldots, \tilde{x}_{m+n})$ will also lie in that interval. Proposition 3 specifies the sense in which this economy displays a high-dimensional indeterminacy, and in which there is a wide variety of perfect-foresight equilibria that display undamped oscillations. In addition, these equilibria can be attained starting from a variety of initial capital stocks. The proposition also demonstrates how private information in the

[14] Here $f^0$ denotes the identity map.

credit market spawns indeterminate equilibria in an overlapping generations economy which otherwise lacks all the features we normally hold responsible for indeterminacy:[15] there is one sector, there are no paper assets, no large income effects, no nonconvexities in production or preferences, no monopolistic competition, nor are there any other significant departures from the key assumptions of Arrow and Debreu—except private information.

It is also straightforward to construct equilibria in which transitions between the two regimes occur stochastically. In particular, let $\Phi_t \in \{0, 1\}$ index the regime, and let

$$(40a) \quad \pi^t_{00} = \text{prob}(\Phi_{t+1} = 0 \mid \Phi_t = 0);$$

$$(40b) \quad \pi^t_{11} = \text{prob}(\Phi_{t+1} = 1 \mid \Phi_t = 1).$$

Then the sequence of matrices

$$(41) \quad \Pi_t = \begin{pmatrix} \pi^t_{00} & 1 - \pi^t_{00} \\ 1 - \pi^t_{11} & \pi^t_{11} \end{pmatrix}$$

induces a (possibly nonstationary) Markov process on the regime index $\Phi_t$. Moreover, competitive equilibria exist for arbitrary sequences $(\pi^t_{00}, \pi^t_{11})$, and the elements of these sequences can depend in essentially any fashion on the history of the economy.

To summarize, in an economy where the steady-state equilibria of each regime lie within the economy's reflective barriers, the expectations of depositors become paramount in determining the type of equilibrium that will be observed. In particular, depositors must form expectations about yields on bank liabilities. Optimistic expectations of high yields lead to Walrasian regimes without credit rationing, and the absence of rationing validates depositors' expectations. Pessimistic expectations of low yields lead to regimes of disintermediation, which forces banks to ration credit. This credit rationing again validates depositors' beliefs. Periodic cycles correspond to alternating, self-confirming waves of optimism and pessimism.

In Section IV, we consider economies where the steady-state equilibria of the two regimes lie outside the economy's reflective barriers. When this occurs, the existence of these barriers limits the scope of depositors' expectations by ruling out persistently optimistic or persistently pessimistic rational expectations equilibria. The result is that *all competitive equilibria must display endogenous volatility,* and that this volatility *cannot* vanish asymptotically.

## IV. Binding Reflective Barriers

We next describe equilibria when neither steady state lies within the economy's reflective barriers. Therefore, in this section we assume that

$$(42) \qquad x^1 < x^1_c < x^0_c < x^0;$$

Evidently, then, neither steady state constitutes a legitimate competitive equilibrium, nor do any monotonic sequences $(x_t)$ that converge to one of the steady states. We now describe what kinds of equilibria can be observed here.

In order to do so, we define two values, $\bar{x}_F$ and $\bar{x}_c$, by the relations

$$\bar{x}_F \equiv \theta x^1_c + (1 - \theta)x^1 \equiv f_1(x^1_c);$$

$$\bar{x}_c \equiv \theta x^0_c + (1 - \theta)x^0 \equiv f_0(x^0_c).$$

It follows from these definitions that $\bar{x}_F$ is simply the point that succeeds $x^1_c$ in regime 1, while $\bar{x}_c$ is the point that succeeds $x^0_c$ in regime 0. These two points represent a floor or ceiling, respectively, on the value of $x_t$ that can be attained in each regime. From equations (28) and (29) it is possible to obtain the following closed-form expressions for $\bar{x}_F$ and $\bar{x}_c$:

$$(43a) \quad \bar{x}_F = \frac{1}{1 - \theta} \log\left[\frac{A(1 - \gamma)}{a}\right];$$

$$(43b) \quad \bar{x}_C = \frac{1}{1 - \theta} \log\left[\frac{A(1 - \gamma)}{a\gamma}\right].$$

[15] See Azariadis (1993 Ch. 26), Roger E. A. Farmer (1993), and Roger Guesnerie and Michael Woodford (1993) for recent surveys of indeterminate equilibria.

An examination of Figure 3(b) should now convince the reader of the following proposition.

PROPOSITION 4: *(i) Every equilibrium sequence of this economy has an upper and a lower turning point, and (ii) both turning points lie inside the invariant set* $[\bar{x}_F, \bar{x}_c]$, *which attracts all equilibrium sequences in finite time.*

In view of the proposition, we now focus our attention on the potential existence of equilibria displaying $(m, n)$ cycles. Such equilibria are defined exactly as in Section III, except that the invariant interval is now $[\bar{x}_F, \bar{x}_c]$ rather than $[x^1, x^0]$.

As in Section III, let $\tilde{x}_1$ and $\tilde{x}_{m+1}$ denote the (unique) fixed points of the compound maps $f_0^m(f_1^n)$ and $f_1^n(f_0^m)$, respectively. Then it is immediately apparent that an $(m, n)$ cycle exists, for fixed $m \geq 1$ and $n \geq 1$, if and only if

$$(44) \qquad \bar{x}_F \leq \tilde{x}_1 < \tilde{x}_{m+1} \leq \bar{x}_c.$$

The next proposition states a necessary condition for (44) to hold for some combination $(m, n)$.

PROPOSITION 5: *Define the parameters* $A_0$, $A_1$, *and* $z$ *by the following relations:*[16]

$$(45a) \qquad A_0 \equiv (\bar{x}_c - x^1)/(x^0 - x^1);$$

$$(45b) \qquad A_1 \equiv (\bar{x}_F - x^1)/(x^0 - x^1);$$

$$(45c) \qquad z \equiv \max\{A_1/(1 - A_1), (1 - A_0)/A_0\}.$$

*Then a deterministic* $(m, n)$ *cycle exists, for some integers* $m \geq 1$ *and* $n \geq 1$, *if* $\theta \geq z$.

[16] Equation (42) implies that $0 < A_1 < A_0 < 1$ holds. The parameters $A_0$ and $A_1$ measure, respectively, the relative distance of the ceiling and floor from the nearest steady state and, hence, reveal how tight reflective barriers are. As $A_0 \to 1$ and $A_1 \to 0$, floors and ceilings become less tight, permitting equilibria to approach the steady states $x^0$ and $x^1$ more closely before reversing direction.

The proof of Proposition 5 appears in Azariadis and Smith (1996b). The proposition establishes a sufficient condition for the existence of an $(m, n)$ cycle.

When $m > (<) n$ holds, an $(m, n)$ cycle is asymmetric in the sense that the number of periods of expansion, $m$, exceeds (is less than) the number of periods of contraction, $n$. When $m = n$, on the other hand, we have a symmetric cycle. We now show that the existence of certain asymmetric $(m, n)$ cycles always implies the existence of at least one—and typically many—symmetric cycles.

PROPOSITION 6: *(a) Suppose that* $A_0 + A_1 \leq 1$ *holds. Then the existence of an* $(m, n)$ *cycle with* $m > n$ *implies the existence of an* $(n, n)$ *cycle. Moreover, if k is any integer with* $k \leq n$, *there exists a* $(k, k)$ *cycle. (b) Suppose that* $A_0 + A_1 \geq 1$ *holds. Then the existence of an* $(m, n)$ *cycle with* $n > m$ *implies the existence of an* $(m, m)$ *cycle. Moreover, if k is any integer with* $k \leq m$, *there exists a* $(k, k)$ *cycle.*

Proposition 6 is proved in Azariadis and Smith (1996b). It states conditions under which the existence of certain $(m, n)$ cycles implies the existence of other (symmetric) cycles. Thus a variety of indeterminacies continues to be observed when these conditions are satisfied.

### A. *Maximally Persistent Cycles*

The cycles discussed in Section III have the property that there are periodic changes in the beliefs of depositors about regimes, and that these belief changes trigger self-fulfilling regime transitions. However, nothing in Section III necessitates these changes in beliefs; in some sense, then, their occurrence may depend on arbitrary historical events. In economies satisfying (42), on the other hand, objective conditions do not permit the beliefs of depositors to remain constant indefinitely. These beliefs must undergo repeated transitions, as must the regime that governs equilibrium behavior. We might ask, then, about the properties of periodic equilibria where depositor beliefs are, loosely speaking, revised as infrequently as possible, and hence where regime switching occurs as infrequently as possible. Our interest in such equilibria leads us to now turn our attention to the existence of what we term *maximally persistent* $(m, n)$ cycles. In particular, a maximally persistent $(m, n)$ cycle has the property that, if there is also a $(p, q)$ cycle, $p + q \leq m + n$ holds. Obviously, a maximally persistent $(m, n)$ cycle exists if (42) holds, and $\theta \geq z$.

Maximally persistent cycles are of interest for two reasons. First, as we have said, these cycles correspond to equilibria with the least volatile depositor expectations that can be observed. Second, if $(m, n)$ is a cycle of maximal persistence, and if $m > (<) n$, all cycles $(m - k, n)$ $[(m, n - k)]$ exist for $k = 1, \ldots, m - n$ $(k = 1, \ldots, n - m)$. Such cycles, of course, exhibit depositor expectations of greater volatility than the $(m, n)$ cycle.

We now state a formal proposition describing maximally persistent cycles. Its proof appears in Azariadis and Smith (1996b).

PROPOSITION 7: *(a) Suppose that inequality (42), $\theta \geq z$, and $A_0 + A_1 \geq 1$ are all satisfied. Let $\hat{s} \geq 1$ denote the largest integer solution to $\theta^s \geq A_1/(1 - A_1)$, and let $\hat{m} \geq 1$ denote the largest integer solution to $\theta^m \geq (1 - A_0)/(1 - A_0\theta^{\hat{s}})$. Then $\hat{m} \geq \hat{s}$ holds, and $(\hat{m}, \hat{s})$ is the maximally persistent $(m, n)$ cycle. (b) Suppose now that (42), $\theta \geq z$, and $A_0 + A_1 \leq 1$ hold. Let $\hat{s} \geq 1$ denote the largest integer solution to $\theta^s \geq (1 - A_0)/A_0$, and let $\hat{n} \geq 1$ denote the largest integer solution to $\theta^n \geq A_1/[1 - (1 - A_1)\theta^{\hat{s}}]$. Then $\hat{n} \geq \hat{s}$ holds, and $(\hat{s}, \hat{n})$ is the maximally persistent $(m, n)$ cycle.*

Note that the proposition asserts that maximally persistent cycles have longer expansions than contractions when $A_0 + A_1 > 1$, because $x^0$ is closer to the ceiling on real activity than $x^1$ is to the floor. Maximally persistent cycles have longer contractions than expansions in the opposite case, and for precisely the opposite reason.

Prior to concluding this section, it is appropriate to mention that $(m, n)$ cycles by no means exhaust the set of possible deterministic periodic equilibria. For example, there may be equilibria with $m$ periods of expansion followed by $n$ periods of contraction, $p$ periods of expansion again, and then $q$ periods of contraction (and so on), with $m \neq p$ and $n \neq q$. Indeed, such equilibria may exist even if $\theta < z$ holds. In addition, stochastic equilibria of the type described in Section III will typically exist. What does *not* exist here are equilibrium sequences that converge to steady states.

## V. CONCLUSIONS

A long tradition in monetary economics holds that the unrestricted operation of financial markets can lead to indeterminacies and excessive fluctuations relative to fundamentals. Previous models that deliver this result, however, have typically depended on the existence of nominal assets [models with nominal assets and production include Ben S. Bernanke and Gertler (1989), Azariadis and Smith (1996a), John H. Boyd and Smith (1997), or Schreft and Smith (1997)], large income effects [Azariadis (1981); Azariadis and Guesnerie (1986)], nonconvexities [Jess Benhabib and Farmer (1994)], monopolistic competition [Jordi Galí (1994); Russell Cooper and Jaoa Ejarque (1994)], or multiple sectors [Nobuhiro Kiyotaki and John Moore (1993); Bencivenga and Smith (1997)]. We have examined a one-sector neoclassical growth model with none of these features, but in which capital investments must be credit financed, and in which credit markets are characterized by the presence of an adverse selection problem. In this model we have shown that the existence of two equilibrium regimes is possible: a Walrasian regime in which incentive constraints are nonbinding, and a regime of credit rationing in which this rationing is necessary to induce self-selection in loan markets. For some range of current capital stocks either regime is consistent with the existence of an equilibrium; therefore, it is possible to observe equilibria in which deterministic or stochastic transitions occur between regimes. Many of these equilibria will display oscillations that do not die out. Moreover, for some configurations of parameters (those considered in Section IV), the *only* equilibria that can be observed display fluctuations that do not vanish asymptotically. In such economies excessive fluctuations are not only possible, but indeed are a necessary feature of any equilibrium.

Relative to much of the existing literature on indeterminancies and endogenous volatility, our analysis contains several new features.

One is the potential for either deterministic or stochastic regime switching. This potential derives from the fact that dynamic equilibria in our model satisfy an unusual, set-valued, discontinuous difference equation which contains two well-defined, partially overlapping domains with distinct structural regimes. At intermediate levels of economic activity transitions between these regimes are possible. Indeed, the resulting equilibria may be indexed by a stochastic process that governs an unobserved regime-switching variable in the manner proposed by James D. Hamilton (1989, 1990). The economic interpretation of this state variable is to regard it as an index of savers' expectations about credit market conditions. An expanding body of evidence suggests that such nonlinear mechanisms accord well with the behavior of many aggregate time series.[17] Yet, while regime-switching models have played an important role in an array of empirical investigations, there has so far been little theoretical work providing any underpinning for them. In addition, much of the theoretical literature studying endogenous volatility has the feature that there are a large set of equilibria, only some of which exhibit fluctuations. We have demonstrated the possibility that such fluctuations not only *can* be observed, but *must* be observed under certain conditions. When these conditions obtain, economic volatility that stems from changing credit market conditions cannot be avoided.

[17] See, for instance, Salih Neftçi (1984), M. Boldin (1990), Paul Beaudry and Gary Koop (1993), Francis Diebold and Glenn Rudebusch (1994), and Simon Potter (1994).

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